

# MATHEMATICS DEPARTMENT CONTEST

Spring semester 2024

The Department of Mathematics invites all Catholic University students to compete, for the fun of it, in a mathematics contest. The contest consists of mathematical problems or puzzles which can be understood by anyone with the usual high school mathematics background. The most successful contestants will be invited to the Mathematics Department end-of-semester party to receive prizes. There will be prizes for the students who solve the most problems and for those who submit the most interesting or original solutions (even if for only one problem).

Submit your solutions by **April 16, 2024** to Dr. Alexander Levin at the Mathematics Department in Aquinas Hall, room 116. They need not be typed but should be legible and should show or explain how you solved the puzzle.

**Problem 1.** There are two types of people on some island: knights who always tell the truth and liars who always lie. Several people who live on the island said that the number of knights on their island is even. All other people from this island said that the number of liars on the island is odd. Determine whether the total number of people on this island is odd or even.

**Problem 2.** Prove that  $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \cdots + \frac{1}{998} - \frac{1}{999} + \frac{1}{1000} < \frac{2}{5}$ .

**Problem 3.** There is a group of eight people such that the sum of their ages is 379. Prove that there are four people in the group such that the sum of their ages is greater than or equal to 189.

**Problem 4.** Prove that if  $m$  is a positive integer, then the number  $400^m - 1$  cannot divide  $2024^m - 1$ .

**Problem 5.** Let  $\triangle ABC$  be a right triangle with the right angle at  $C$ . Suppose that the height to the hypotenuse  $CH$  divides the bisector of the angle  $B$  into two equal parts. Find the angle  $A$ . (Give your answer in degrees or in radians.)

**Problem 6.** There were several (more than two) teams in a volleyball league. Each team played exactly one game against each other team. For each game, the winning team got 1 point and the losing team got 0 points. There were no draws. After all games, the coefficient of a team  $A$  was computed as the total number of points received by the teams that had lost to  $A$ . It turned out that all teams had the same coefficient. Prove that all teams received the same number of points.